THEORETICAL COMPETITION

9th Asian Physics Olympiad

Ulaanbaatar, Mongolia (April 22, 2008)

Problem 3. How does a superluminal object look like?

Can a body move faster than the speed of light? The answer is “No” if the object is moving in the vacuum. But the answer can be “Yes”, if we deal with the phase speed of light in an optically dense medium with refractive index of n ($n = c/u$, where u is the speed of light in the medium, and c is the speed of light in the vacuum).

We say a body is superluminal, if $u < v < c$, where v is the velocity of the body. One of the well known examples of the superluminal body is a charged particle generating Cherenkov radiation.

Throughout the problem we will deal with a superluminal body of constant velocity v in an optical medium without dispersion. u is the velocity of light in the medium.

For the simplicity, we introduce a notation $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$ and an angle θ given by $\cos\theta = u/v$

and $\tan\theta = \sqrt{\frac{v^2}{u^2} - 1}$.

1. Radiating superluminal particle

As shown in Fig.1, a radiating particle is moving along the x-axis with a constant velocity v ($v > u$).

An observer M is located at the distance d from x-axis.

We choose the point nearest to the observer as the point O, the origin on the x-axis. The time when the particle actually passes over the point x=0 is taken to be t=0.

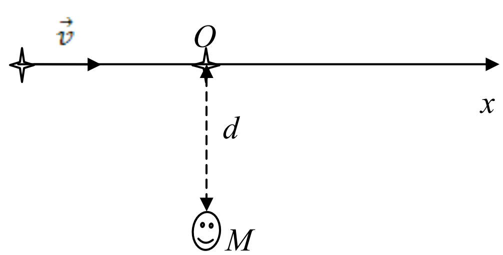

Figure 1

(1)Suppose the light radiated at the given time t' is observed at time t. Express t in terms of d, t', u and v. (1.0 point)

(2)At time $t = t_0$, the observer first sees the particle at position x'_0. Find the apparent position x'_0 and the observed time t_0 for this first appearance in terms of d, v and θ.
(2.0 points)

(3) Find the apparent position(s) x' of the particle for any given time t. Write your answer in terms of v, θ, t and t_0. (2.0 points)

(4) Find the apparent velocity(s) v'(t) of the particle for any given time t. Write your answer in terms of v, θ, t, and t_0. (1.0 point)

(5) Find the apparent velocity(s) v' of the first appearance of the particle. (0.2 points)

(6) Find the apparent velocity(s) v' of the particle at infinite distances from the origin, O. Write your answer in terms of v and u. (0.2 points)

(7) Sketch the graph of the apparent velocity v' versus time t, indicating clearly asymptotic values of the apparent velocity. (1.0 point)

(8) Can an apparent velocity exceed the light speed in the vacuum, i.e. $v' > c$? (0.2 points)

2.Radiating linear object

Consider a linear object, radiating light and moving along the x-axis. The length of the linear object is L in the rest frame of the object.

A. Parallel movement

In this section, we assume that the radiating linear object moves longitudinally along x-axis as shown in Fig.2.

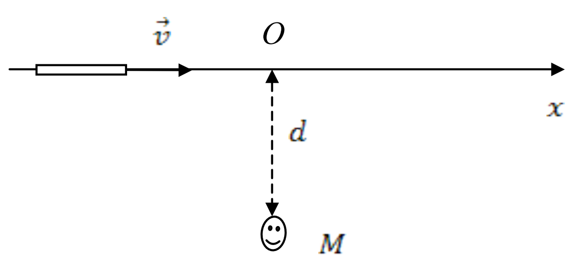

Figure 2

(9) Determine the time interval of complete appearance of the whole linear object from the first appearance of its front point. Write your answer in terms of L, γ and v. (0.3 points)

(10) Determine the apparent length(s) of the object at the moment of its complete appearance. Write your answer in terms of d, L, θ and γ. (0.4 points)

B. Perpendicular movement

In this section, we assume that the radiating linear object moves perpendicularly along x-axis as shown in Fig.3. Let the observer be located at the origin of x-axis $(d = 0)$. The object is symmetrical with respect to x-axis.

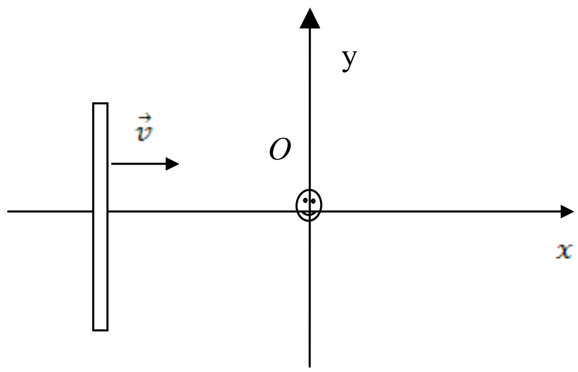

Figure 3

(11) Show that for a given time t, the apparent form of this object is an ellipse or part(s) of an ellipse. (0.7 points)

Find the following quantities and express them in terms of v, θ, and t.

(12) Find the position x_c of the centre of symmetry of the ellipse for a given time t in terms of v, θ and t. (0.5 points)

(13) Determine the lengths of the semi-major and semi-minor axes of the ellipse for a given time t in terms of v, θ and t. (0.5 points)