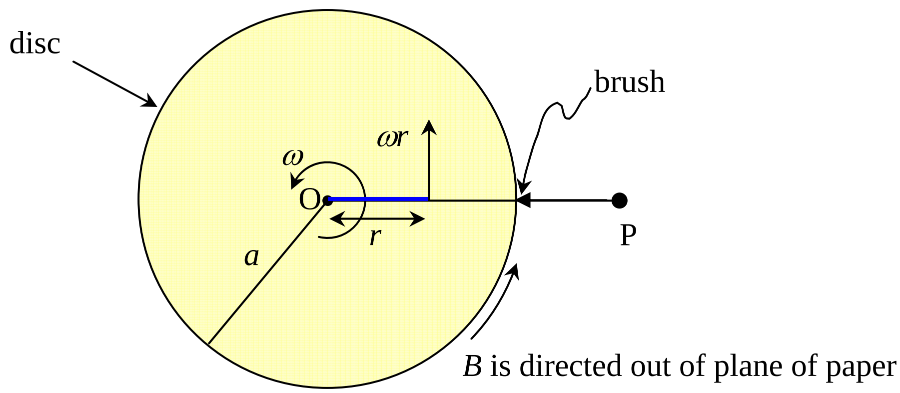

A Self-excited Magnetic Dynamo

2.1) $$L\frac{d}{dt}i + Ri \;=\; \mathcal{E}$$ …………………(i) (1.0 point)

2.2) If the length ℓ of solenoid is much greater than its diameter, then the magnetic flux density in the inside mid-section of the solenoid is given by

$$B \;=\; \frac{\mu_0 N i}{\ell}$$ ………………(ii) (1.5 points)

2.3)



The electric field intensity (E) at distance r from the centre of the axle is

$$E = B\omega r$$

pointing towards the rim of the disc. The induced e.m.f. $(\mathcal{E})$ between terminals P and Q is given by

$$\mathcal{E} \;=\; \int_{r=0}^{a} B\omega r dr \;=\; \frac{1}{2}B\omega a^2 \;=\; \frac{\mu_0 N i \omega a^2}{2\ell}$$ …………………(iii) (2.0 points)

2.4) By combining the results in 2.1) and 2.3) we get

$$L\frac{d}{dt}i + Ri \;=\; \left(\frac{\mu_0 N a^2 \omega}{2\ell}\right)i$$

$$\frac{d}{dt}i \;=\; +\frac{1}{L}\left(\frac{\mu_0 N a^2 \omega}{2\ell} - R\right)i \;\equiv\; +\gamma i$$ (0.5 point)

$$\therefore \qquad i(t) \;=\; i(0)e^{\gamma t}$$ …………………(iv) (1.0 point)

where $\gamma \equiv \frac{1}{L}\left(\frac{\mu_0 N a^2 \omega}{2\ell} - R\right)$

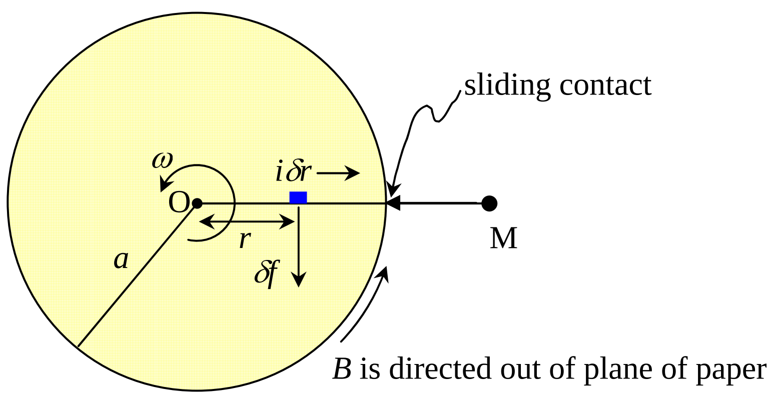
2.5) In order that the current $i(t)$ will grow, the value of γ must be positive otherwise the current will gradually decay.

$$\frac{\mu_0 N a^2 \omega}{2\ell} - R \geq 0 \qquad \text{(1.5 points)}$$

$$\omega_{\min} = \frac{2\ell R}{\mu_0 N a^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\text{(v)} \qquad \text{(0.5 point)}$$

2.6)

Method 1



At the instant t the current is given in 4) as $i(t) = i(0)e^{\gamma t}$.

The magnetic force δf on the current element $i\delta r$ is $\delta f = Bi\delta r$. (1.0 point)

The torque $\delta\tau = r\delta f = Bir\delta r$ opposes the rotation of the disc.

The total torque $\tau = \int_{r=0}^{r=a} Birdr = \frac{1}{2}Bia^2$

$$\tau = \frac{\mu_0 N a^2}{2\ell} i^2 = \frac{\mu_0 N a^2}{2\ell} i^2(0) e^{+2\gamma t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\text{(vi)} \qquad \text{(1.0 point)}$$

In order to maintain the angular velocity of the disc at a steady value we must apply a turning torque of equal magnitude and of opposite direction to that of (vi).

$\boxed{\textbf{Method 2}}$

$$\tau\omega = i^2R + \frac{d}{dt}\left(\text{magnetic energy in solenoid}\right) \qquad \text{(1.0 point)}$$

$$= i^2R + L\frac{di}{dt}i$$

$$\tau = \frac{\mu_0 N a^2}{2\ell} i^2 \quad = \quad \frac{\mu_0 N a^2}{2\ell} i^2\left(0\right)e^{+2\gamma t} \qquad \text{(1.0 point)}$$

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